

Cambridge International Examinations

Cambridge Ordinary Level

ADDITIONAL MATHEMATICS

4037/22

Paper 2 May/June 2016

MARK SCHEME
Maximum Mark: 80

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Page 2	Mark Scheme S		Paper
	Cambridge O Level – May/June 2016	4037	22

Abbreviations

awrt answers which round to cao correct answer only

dep dependent

FT follow through after error isw ignore subsequent working

oe or equivalent

rot rounded or truncated

SC Special Case soi seen or implied

www without wrong working

Q	uestion	Answer	Marks	Guidance
1	(i)	$(2k)^2 - 4(1)(4k - 3)$ [< 0] Correct completion to given inequality $k^2 - 4k + 3 < 0$ isw	M1 A1	clear attempt at $b^2 - 4ac$
	(ii)	Critical values 1 and 3 soi $1 < k < 3$ as final answer	M1 A1	May be implied by incorrect inequalities
2	(i)	Clear attempt at quotient rule or equivalent product rule $\left[\frac{dy}{dx} = \right] \frac{14}{(3-x)^2}$ or $\left[\frac{dy}{dx} = \right] \frac{14}{x^2 - 6x + 9}$ cao or correct simplified equivalent	M1 A1	allow recovery from bracketing errors or omissions if implied in correct work to the correct answer
	(ii)	$[y = 9] x = 2$ $\frac{0.07}{\delta x} \approx \left(their \frac{dy}{dx} \Big _{x=2} \right) oe$ $0.005 oe$	B1 M1 A1	condone $\frac{0.07}{\delta x} = \left(their \frac{dy}{dx}\Big _{x=2}\right)$ not from wrong working; answer only does not score
3		Any one of: $\begin{bmatrix} {}^{6}C_{0} \times {}^{7}C_{3} + {}^{6}C_{1} \times {}^{7}C_{2} \\ \text{or } 35 + 126 \\ \text{or } {}^{13}C_{3} - {}^{6}C_{2} \times {}^{7}C_{1} - {}^{6}C_{3} \\ \text{or } 286 - 105 - 20 \end{bmatrix}$	M2	M1 for $\begin{bmatrix} {}^{6}C_{0} \times \end{bmatrix} {}^{7}C_{3}$ or ${}^{6}C_{1} \times {}^{7}C_{2}$ or ${}^{13}C_{3} - {}^{6}C_{2} \times {}^{7}C_{1}$ or ${}^{13}C_{3} - {}^{6}C_{3}$ or ${}^{6}C_{2} \times {}^{7}C_{1} + {}^{6}C_{3}$ or for the numerical equivalent of one of these calculations
		161	A1	If M0 then B3 for answer only of 161

Page 3	Mark Scheme S		Paper
	Cambridge O Level – May/June 2016	4037	22

Question	Answer	Marks	Guidance
4 (i)	$2(2)^3 - 3(2)^2 + 2q + 56 = 0$ with one correct interim step leading to $q = -30$	B1	allow for only $16 - 12 + 2q + 56 = 0$ q = -30
			NB = 0 must be seen or may be implied by e.g. $-60 = 2q$ or $60 = -2q$;
			or convincingly showing $2(2)^3 - 3(2)^2 - 30(2) + 56 = 0$; allow for only 16 - 12 + 2(-30) + 56 = 0
			or correct synthetic division at least as far as $ \begin{array}{c ccccccccccccccccccccccccccccccccccc$
			$\begin{vmatrix} 2 & 1 & q+2 & 0 \\ \text{then } q = -30 \end{vmatrix}$
(ii)	$2x^{2} + x - 28$ $(x-2)(2x-7)(x+4)$	B2 M1	B1 for any two terms correct For factorising the correct equation; condone = 0; condone $(2x-7)(x+4)$ only for M1 but for A1 must see all 3 factors in this part; do not allow $\left(x-\frac{7}{2}\right)$
	x = 2, x = -4, x = 3.5 oe	A1	not from wrong working; answers only do not score
5 (i)	(2, 8)	B1, B1	
(ii)	$\frac{their8 - 0}{their2 - p} = -2 \text{ or better}$	M1	Condone $\frac{their8 - 0}{their2 - p} = \frac{-1}{their \text{ gradient } AB} \text{ oe}$
	[p =] 6	A1	F Brancher 12

Page 4	Mark Scheme S		Paper
	Cambridge O Level – May/June 2016	4037	22

Question	Answer	Marks	Guidance
(iii)	$[MB =] \sqrt{(6 - their 2)^2 + (10 - their 8)^2}$	M1	implied by $[MB =]\sqrt{20}$ or
	or $\left[\frac{1}{2}AB = \right] \frac{1}{2}\sqrt{(6-2)^2 + (10-6)^2}$		$\left[\frac{1}{2}AB = \right]\frac{1}{2}\sqrt{80} \text{ e.g. 4.47},$
	or $[MC =] \sqrt{(their 2 - their p)^2 + (their 8 - 0)^2}$		or $[MC =]\sqrt{80}$ or e.g. 8.94 or 63.4° or equivalents
	$[MC = J\sqrt{(their2 - their p)^{-} + (their8 - 0)^{-}}$ soi		
	or tan[] = $\frac{8}{4}$ soi		
	or $4.47^2 = 8.94^2 + 10^2 - 2(8.94)(10)\cos[]$		
	or $8.94^2 = 10^2 + 10^2 - 2(10)(10)\cos[]$		
	$\sin^{-1}\left(\frac{\sqrt{20}}{10}\right)$ oe soi	M1	$\operatorname{or} \cos^{-1} \left(\frac{\sqrt{80}}{10} \right)$
			or $\tan^{-1}\left(\frac{\sqrt{20}}{\sqrt{80}}\right)$
			or $\tan^{-1}\left(\frac{4}{8}\right)$
			or $90 - \tan^{-1}\left(\frac{8}{4}\right)$
			or equivalent complete correct method; implies first M1
	26.56 to 26.6° or 0.4636 to 0.464 rads cao	A1	Not from wrong working
6 (i)	Valid explanation	B1	e.g. arc length is greater than the radius or 7 is greater than 5
(ii)	$7 = 5\theta$ $\theta = 1.4 \text{ oe}$	M1 A1	implies M1
(iii)	$\frac{1}{2} \times 5^2 \times their 1.4$ oe	M1	
	17.5oe	A1	

Page 5	Mark Scheme	Syllabus	Paper
	Cambridge O Level – May/June 2016	4037	22

Question	Answer	Marks	Guidance
(iv)	[triangle area =] $\frac{1}{2} \times 5^2 \times \sin their$ 1.4 or 12.3 to 12.32 or for [$\frac{1}{2} \times \text{base} \times \text{height=}$] $\frac{1}{2} \times 6.4[4] \times 3.8[2]$ oe	M1	may be embedded in a difference calculation
	5.18 to 5.2 inclusive	A1	implies M1
7 (i)	$ \begin{pmatrix} 12 & 15 \\ 9 & 6 \end{pmatrix} + \begin{pmatrix} 4 & 2 \\ 1 & 3 \end{pmatrix} soi $	M1	if no method shown, may be implied by their answer with at least 2 correct elements
	$ \begin{pmatrix} 16 & 17 \\ 10 & 9 \end{pmatrix} $	A1	
(ii)	$det A = 4 \times 2 - 3 \times 5 = -7$ or $det B = 4 \times 3 - 2 \times 1 = 10$	B1	allow for e.g. $(4 \times 2 - 3 \times 5) \times (4 \times 3 - 2 \times 1) = -70$
(iii)	$\mathbf{AB} = \begin{pmatrix} 21 & 23 \\ 14 & 12 \end{pmatrix}$ $\det(\mathbf{AB}) = 21 \times 12 - 23 \times 14 = -70$ $\frac{1}{their} \det \mathbf{AB} \times their \begin{pmatrix} 12 & -23 \\ -14 & 21 \end{pmatrix} \text{ isw}$	B2 B1	or $\det \mathbf{A} = 8 - 15 = -7$ or $\det \mathbf{B} = 12 - 2 = 10$ or $\mathbf{B1}$ for two elements correct allow for $\det(\mathbf{AB}) = 252 - 322 = -70$ For full marks must conclude that $\det \mathbf{AB} = \det \mathbf{A} \times \det \mathbf{B}$ or show the product $-7 \times 10 = -70$ otherwise max 3 marks correct or correct \mathbf{FT} ; \mathbf{FT} their \mathbf{AB} and their non-zero $\det \mathbf{AB}$; their \mathbf{AB} must be an attempt at a matrix product e.g. $\begin{pmatrix} 16 & 10 \\ 3 & 6 \end{pmatrix}$ $\mathbf{B1}$ for $\frac{1}{their} \det \mathbf{AB} \times their$
			or for $k \times their \begin{pmatrix} 12 & -23 \\ -14 & 21 \end{pmatrix}$

Page 6	Mark Scheme S		Paper
	Cambridge O Level – May/June 2016	4037	22

Question	Answer	Marks	Guidance
8	Eliminates y e.g. $4 + \frac{5}{15x + 10} + \frac{3}{x} = 0$ or eliminates x e.g. $4 + \frac{5}{y} + \frac{3}{(y - 10)/15} = 0$	M1	allow even after incorrect rearrangement of the equation of the curve (dependent on resulting equation still in terms of x and y); condone substitution of e.g. $\frac{y+10}{15}$
	Rearrange to a 3-term quadratic $60x^2 + 90x + 30 = 0$ oe or $4y^2 + 10y - 50 = 0$ oe	M1 A1	condone sign slips/arithmetic slips
	Factorise or solve 3-term quadratic	M1	
	$x = -\frac{1}{2}, x = -1 \text{ isw}$	A1	or $y = 2\frac{1}{2}$, $y = -5$
	$y = 2\frac{1}{2}$, $y = -5$ isw	A1	or $x = -\frac{1}{2}$, $x = -1$
			If final A marks not awarded then A1 for a correct <i>x</i> , <i>y</i> pair
9 (a)	$\frac{x^2}{2} + x - \frac{1}{x} (+c) \text{isw}$	В3	B1 for each term allow $\frac{x^2}{2} + x + \frac{x^{-1}}{-1}(+c)$ isw for B3
(b) (i)	$k\cos(5x + \pi) \text{ where } k < 0$ or $\frac{\cos(5x + \pi)}{5}$	M1	
	$\frac{-\cos(5x+\pi)}{5}(+c)$	A1	
(ii)	$\frac{-\cos(5(0) + \pi)}{5} - \frac{-\cos(5(-\pi/5) + \pi)}{5}$ $\operatorname{or} \frac{-\cos(\pi)}{5} - \left(\frac{-\cos(0)}{5}\right)$	M1	correct substitution of the given limits into their expression of the form $k\cos(5x+\pi)$, dep on M1 in (b)(i)
	0.4 oe	A1	answer only does not score
10 (a)	2 = p - q and 14 = 4p - 2q oe $p = 5$ $q = 3$	M1 A1 A1	
(b)	Factorise $10^{2x} - 2(10^x) - 24 = 0$ or factorise $u^2 - 2u - 24 = 0$	M1	or applies the formula or completes the square
	$10^{x} = 6$ $x = \lg 6 \text{cao as final answer}$	A1 A1	ignore $10^x = -4$ for this mark or exact equivalent

Page 7	Mark Scheme	Syllabus	Paper
	Cambridge O Level – May/June 2016	4037	22

Question	Answer	Marks	Guidance
(c)	$\frac{x+1}{x} = 2^3 \text{ oe www}$	M2	combines logs and anti-logs or B1 for one correct log move
			$e.g. \log_2\left(\frac{x+1}{x}\right) = 3$
			or $\log_2(x+1) - \log_2(x) = \log_2 8$
	1		or $\log_2(x+1) - \log_2(x) = 3\log_2 2$
	$x = \frac{1}{7}$ or 0.143 or 0.1428 to 0.1429	A1	
11 (a)	Valid method	M1	Completing the square as far as
			e.g. constant $-\left(x-\frac{1}{2}\right)^2$
			or calculus as far as $1 - 2x = 0$
			or finding roots $x = 0$ and $x = 1$ and using symmetry soi
	when $x = \frac{1}{2}$	A1	Implies M1 if not clearly from wrong working
	[greatest value =] $\frac{1}{4}$	B1	
(b)	Valid comment e.g. when $x \ge 1$, f' is always	B1	Allow e.g. a sketch with a comment such as the curve is one-one [when $x \ge 1$]
	decreasing		or e.g. the curve is one-one when $x > \frac{1}{2}$
(c) (i)	$k(10) = 8 \text{ or } 5 + \sqrt{10 - 1} = 8 \text{ or stating}$ h(8)	M1	$or[hk(x) =] lg(7 + \sqrt{x-1})$
	$h(8) = 1 \text{ or } \lg(8+2) = 1 \text{ cao}$	A1	$[hk(10) =] lg(7 + \sqrt{10 - 1}) = 1$
(ii)	$\left(y-5\right)^2 = x-1$	M1	$\operatorname{or}(x-5)^2 = y-1$
	$k^{-1}(x) = (x-5)^2 + 1$ isw	A1	
	or $k^{-1}(x) = x^2 - 10x + 26$ isw 5 < x < 15	B1, B1	B1 for $5 < x$ oe and B1 for $x < 15$ oe
			allow (5, 15); one mark for each limit of the interval;
			if B0 then SC1 for $5 \le x \le 15$ or '5 to 15' or [5, 15] etc.
	$1 < k^{-1}(x) < 101$	B1	allow (1, 101)

Page 8	Mark Scheme	Syllabus	Paper
	Cambridge O Level – May/June 2016	4037	22

Question	Answer	Marks	Guidance
12 (i)	$8(1-\cos^2 A) + 2\cos A = 7$ or better Solves or factorises <i>their</i> 3-term quadratic in $\cos A$	B1 M1	
	60, 104.477 rounded or truncated to 1 dp or more;	A2	with no extras in range; not from clearly wrong working but allow recovery from minor slips or A1 for either, ignoring extras
(ii)	$\sin(3B+1) = 0.4 \text{ soi}$	B1	may be implied by $\frac{1}{\sin(3B+1)} = 2.5$
	[3B + 1 =] 0.41 or better	M1	implies B1
	0.577, 1.9[0], 2.67 or 0.57669, 1.89823, 2.67108 rounded or truncated to 4 or more sf	A2	with no extras in range; or A1 for any one correct ignoring extras If M0 then B2 for all 3 correct angles found or B1 for 1 or 2 correct angles found